AA214 – Problem Set Split/Factorization

1. *Multisplit Schemes.* Starting with the generic ODE

$$\frac{\partial \overline{u}}{\partial t} = [A]\overline{u} + \overline{f}$$

Splitting $[A] = [A_1 + A_2 + A_3 + A_4]$ and applying Implicit Euler time differencing

$$\frac{\overline{u}_{n+1} - \overline{u}_n}{h} = [A_1]\overline{u}_{n+1} + [A_2]\overline{u}_{n+1} + [A_3]\overline{u}_{n+1} + [A_4]\overline{u}_{n+1} + \overline{f}$$

(a) Write the Factored Delta form of Implicit Euler time differencing. What is the error term?

(b) Instead of making all the split terms implicit, leave two explicit

$$\frac{\overline{u}_{n+1} - \overline{u}_n}{h} = [A_1]\overline{u}_{n+1} + [A_2]\overline{u}_n + [A_3]\overline{u}_{n+1} + [A_4]\overline{u}_n + \overline{f}$$

Write the resulting Factored Delta form implicit algorithm and define the error terms.

(c) The scalar representative equation is

$$\frac{\partial u}{\partial t} = (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)u + a$$

For both the unfactored and factored implicit scheme of Problem (1a), do the scalar analysis for the $\sigma - \lambda$ relation and comment on the resulting stability, convergence, and accuracy.

(d) Repeat (c) for the explicit-implicit scheme of Problem (1b).

2. *System Splitting, Plus-Minus Splitting.* Consider the system

$$\frac{\partial u}{\partial t} - \frac{\partial v}{\partial x} = 0 \quad , \quad \frac{\partial v}{\partial t} - \frac{\partial u}{\partial x} = 0$$

(a) Write in matrix vector form and define terms.

$$\frac{\partial \overline{q}}{\partial t} + [A]\frac{\partial \overline{q}}{\partial x} = 0$$

(Hint: define $\overline{q} = \begin{pmatrix} u \\ v \end{pmatrix}$).

(b) Find the eigenvalues and eigenvectors of $[A]$ and form a Plus-Minus Flux Vector Splitting. Define all the terms, vectors and matrices. (That is, $E^{\pm}, [A]^{\pm}$).

(c) For Implicit Euler time differencing, write down the unfactored and factored delta form and comment on why you would use on and not the other.